Q1a

a) $(0.8)^x = e^{\ln 0.8^x}$

$= e^{x \ln 0.8}$

$= \boxed{e^{-0.223x}}$

$a = e^{\ln a}$

Q1b

b) i) $\left(\frac{2}{3}\right)^{4t+1} = e^{\ln\left(\frac{2}{3}\right)^{4t+1}}$

$a = e^{\ln a}$

$= e^{(4t+1)\ln\frac{2}{3}}$

$= e^{4t\ln\frac{2}{3}} e^{\ln\frac{2}{3}}$

$= \frac{2}{3}e^{4t\ln\frac{2}{3}}$

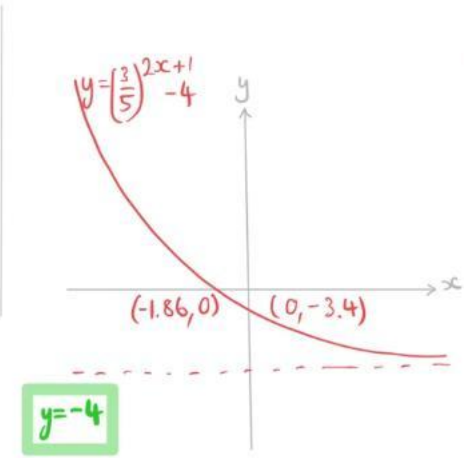
$k = 4\ln\frac{2}{3} = -1.62$

$\boxed{\frac{2}{3}e^{-1.62t}}$

ii) since k, $\boxed{-1.62 < 0 \rightarrow \text{exponential decay}}$

iii) $t=0$ $\quad Ae^{k(0)} = A = \boxed{\frac{2}{3}}$

Q2



(2) rewrite term containing x

$$e^{\ln\left(\frac{3}{5}\right)^{2x+1}} = e^{(2x+1)\ln\frac{3}{5}}$$
$$= e^{2x\ln\frac{3}{5}} e^{\ln\frac{3}{5}}$$
$$= \frac{3}{5} e^{\left(2\ln\frac{3}{5}\right)x}$$
$$= \frac{3}{5} e^{-1.02x}$$

(3) so eqn can be written as:

$$y = \frac{3}{5} e^{-1.02x} -4$$

stretch in y axis by $\frac{3}{5}$

$-1.02<0$ $\therefore$ exponential decay

stretch in x axis by 1.02

translating by -4 in y axis

(4) consider transformations on curve $y=e^{-x}$...

(5) POINTS OF INTERSECTION

at $x=0$

$$y = \frac{3}{5} e^{-1.02(0)} - 4$$
$$= \frac{3}{5} - 4$$
$$= -3.4$$

at $y=0$

$$0 = \frac{3}{5} e^{-1.02x} - 4$$
$$\frac{20}{3} = e^{-1.02x}$$
$$-1.02x = \ln\frac{20}{3}$$
$$x = -1.86$$

Q3A

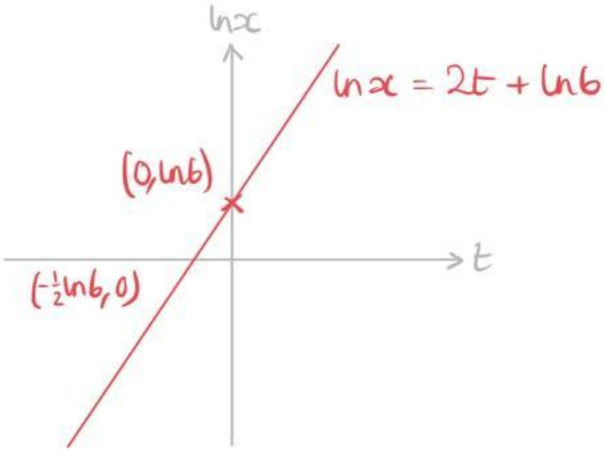
a) $e^{\ln x} = e^{2t+\ln 6}$

$$x = e^{2t} e^{\ln 6}$$

$$\boxed{x = 6e^{2t}}$$

Q3B

b)



when $\ln x = 0$

$$0 = 2t + \ln 6$$

$$t = -\frac{1}{2}\ln 6 = -0.896$$

Q4A

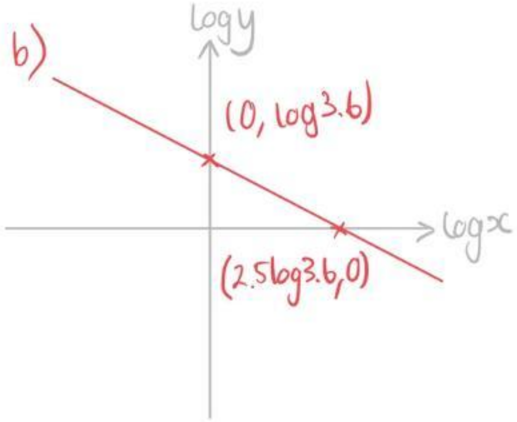
a) $\log y = \log 3.6x^{-0.4}$

$$= \log 3.6 + \log x^{-0.4}$$

$$\boxed{\log y = \log 3.6 - 0.4\log x}$$

Q4b

b)



log x axis intercept:

$\log y = 0 = \log 3.6 - 0.4 \log x$

$\log x = \dfrac{\log 3.6}{0.4}$

$= 2.5 \log 3.6$

Q5a

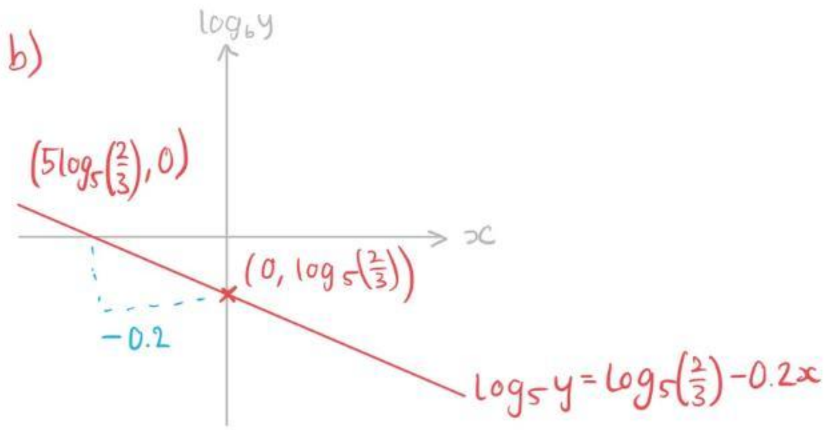
a) $\log_5 y = \log_5 \left(\frac{2}{3}\right) 5^{-0.2x}$

$= \log_5 \left(\frac{2}{3}\right) + \log_5 5^{-0.2x}$

$\boxed{\log_5 y = \log_5 \left(\frac{2}{3}\right) - 0.2x}$

Q5b

b)



x-axis intercept:

$$\log_6 y = 0 = \log_5\left(\frac{2}{3}\right) - 0.2x$$

$$x = \frac{\log_5\left(\frac{2}{3}\right)}{0.2} = 5\log_5\left(\frac{2}{3}\right)$$

Q6a

a) $\boxed{\text{Population will increase exponentially} \quad \therefore k \text{ is positive.}}$

Q6b

b) at $m=8$, $A = 16 \times 1.5 = 24$

initial population

50% increase

$$24 = 16e^{8k}$$

$$\frac{3}{2} = e^{8k}$$

$$\ln\frac{3}{2} = 8k$$

$$k = \frac{1}{8}\ln\frac{3}{2} = \boxed{0.0507} \text{ (3sf)}$$

Q6c

c) Sub $A = 3000$ into model

$$3000 = 16e^{0.0507m}$$

$$\ln\frac{3000}{16} = 0.0507m$$

$$m = \frac{\ln\frac{3000}{16}}{0.0507} = \boxed{\text{103 months}} \text{ (3sf)}$$

Q7a

a) i) $T = \text{initial temperature} + \text{rate of change of temp} \times t$

$85 + \frac{\Delta T}{\Delta t} \times t$

$\frac{\Delta T}{\Delta t} = \frac{50-85}{7} = -5$

$T = 85 - 5t$

ii) A: initial temp $= 85°C$

$50 = 85e^{-7k}$

$\ln \frac{50}{85} = -7k$

$k = \frac{1}{-7} \ln \frac{50}{85} = 0.0758$ (3sf)

$\boxed{T = 85e^{-0.0758t}}$

Q7b

b) At $t = 3$

i) $\frac{dT}{dt} = -5$

ii) $\frac{dT}{dt} = 85(-0.0758)e^{-0.0758(3)}$

$= -5.13$

rate of cooling is faster in the exponential model

Q7c

c) The manufacturer claims the drink is kept 'warm' for up to 7h, but 5h is less than 7h. However, the user may have opened (and left open) the flask for sometime during the 7hs whilst drinking from it.

Q8a

a) Even with the allowance of negative time, the rocket would not be accelerating before lift off.

Q8b

b) Form simultaneous eqns using pairs of values for A and t.

At $t=5$, $A=12$ At $t=20$, $A=50$

① $12 = Re^{5k}$ ② $50 = Re^{20k}$

① ÷ ②

$$\frac{12}{50} = \frac{\cancel{R}e^{5k}}{\cancel{R}e^{20k}} = e^{5k-20k} = e^{-15k}$$

$$k = -\frac{1}{15}\ln\frac{12}{50} \quad \boxed{k = 0.0951} \quad (3sf)$$

Sub k into eqn ① (or ②)

$$12 = Re^{5(0.0951)}$$

$$\boxed{R = 7.46} \quad (3sf)$$

Q8c

c) $t=5, A=12$ $t=20, A=50$

① $12=R+c5$ ② $50=R+20c$

② − ①

$$50-12=\cancel{R}+20c-(\cancel{R}+5c)$$
$$38=15c$$
$$c=\frac{38}{15}$$

Sub c into ① (or ②)

$$12=R+\left(\frac{38}{15}\right)5$$
$$R=-\frac{2}{3}$$

LINEAR MODEL $A=-\frac{2}{3}+\frac{38}{15}t$

When $t=0$

$$A=-\frac{2}{3}+0=-\frac{2}{3}$$

The linear model produces a negative initial acceleration which is unrealistic.

Q9a

a) At $t=0$

$$m=M_0e^{-k(0)}=M_0$$

M_0: initial mass of C-14 in an object

Q9B

b) At $t = 5700$ $m = \frac{M_0}{2}$

$$\frac{\cancel{M_0}}{2} = \cancel{M_0} e^{-5700k}$$

$$\ln \frac{1}{2} = -5700k$$

$$k = \frac{-\ln \frac{1}{2}}{5700}$$

$$= \frac{\ln \left(\frac{1}{2}\right)^{-1}}{5700}$$

$$\boxed{k = \frac{\ln 2}{5700}}$$

Q9C

c) $M_0 = 200$ $t = 20000$

$$m = 200 e^{-\frac{\ln 2}{5700}(20000)}$$

$$= \boxed{17.6g}$$ (nearest gram)

Q9D

d) $m = 3\times10^{-6}$ At $t = 5700 \pm 40$

$M_0 = 1\times10^{-2}$ $m_{\frac{1}{2}} = \frac{3\times10^{-6}}{2} = 1.5\times10^{-6}$

Find upper and lower values for k using $t = 5700 \pm 40$

UB (+40)	LB (−40)
$k = \frac{\ln 2}{5740}$	$k = \frac{\ln 2}{5660}$

Make t the subject of the model

$$\frac{m}{M_0} = e^{-kt}$$

$$\ln\frac{m}{M_0} = -kt$$

$$t = -\frac{1}{k}\ln\frac{m}{M_0} = -\frac{1}{k}\ln\frac{3\times10^{-6}}{1\times10^{-2}} = -\frac{1}{k}\ln(3\times10^{-4})$$

Find t using upper and lower values of k

UB	LB
$t = \frac{-1}{\frac{\ln 2}{5740}}\ln(3\times10^{-4})$	$t = \frac{-1}{\frac{\ln 2}{5660}}\ln(3\times10^{-4})$
$= 67173$	$= 66237$
$\boxed{t_{UB} = 67000y}$ (2sf)	$\boxed{t_{LB} = 66000y}$ (2sf)

Q10A

a)

t	0	1.5	3	4.5	6
$\log_2 N$	6.91	7.57	8.49	9.41	10.26

$y = mx + c$

$\log_2 N = mt + C$

$m = \dfrac{\Delta \log_2 N}{\Delta t} = \dfrac{\log_2 1230 - \log_2 120}{6 - 0}$

$= 0.560$ (3sf)

y-intercept

$\log_2 120 = 6.91$ (3sf)

Line of best fit: $\log_2 N = 0.560t + \log_2 120$

Rewrite model in the form of the line of best fit eqn, then compare the two, to find estimates for the unknowns.

$\log_a N = \log_a N_0 a^{kt}$

$= \log_a N_0 + \log_a a^{kt}$

$\log_a N = \log_a N_0 + kt$

$\boxed{\begin{aligned} a &= 2 \\ N_0 &= 120 \\ k &= 0.560 \end{aligned}}$

Q10B

b) At $t = 12$

$N = 120(2)^{0.560(12)}$

$= \boxed{12600}$ (3sf)

12 is far away from the range of the data used to produce the model. We have extrapolated to get this prediction, therefore it's not reliable.

Q11A

a) 1) Draw line of best fit.

2) Find eqn for line of best fit.

$$\ln D = mt + c$$

$$m = \frac{\Delta \ln D}{\Delta t} = \frac{2-3.5}{3-0} = -\frac{1}{2}$$

$$c = 3.5 = \ln D$$

$$D = e^{3.5} = 33.1 \; (3sf)$$

Line of best fit: $\ln D = -\frac{1}{2}t + \ln 33.1$

3) Rewrite model in the same form as the line of best fit.

$$\ln D = \ln Ae^{-kt}$$

$$= \ln A + \ln e^{-kt}$$

$$\ln D = \ln A - kt$$

4) Compare constants.

$$\ln D = -\frac{1}{2}t + \ln 33.1$$

$$\ln D = -kt + \ln A$$

$$\boxed{\begin{array}{l} A = 33.1 \\ k = \frac{1}{2} \end{array}}$$

Q11B

b) $\frac{dD}{dt} = A(-k)e^{-kt} = -12$ "rate of decrease"!

$= 33.1\left(-\frac{1}{2}\right)e^{-\frac{1}{2}t} = -12$

$e^{-\frac{1}{2}t} = \frac{24}{33.1}$

$-\frac{1}{2}t = \ln\frac{24}{33.1}$

$t = -2\ln\frac{24}{33.1} = 0.644\,h$ (3sf)

$= 0.644 \times 60$ mins in 1h

$\boxed{t = 39\text{mins}}$

Q12A

a) $a = 1$, $\log P = 3.74$

$10^{3.74} = P$

$10^{3.74} = P_1(1)^k$ Sub a & P into model

$P_1 = 10^{3.74}$

$\boxed{P_1 = £5495}$ (nearest pound)

$a = 2$, $\log P = 3.86$

$10^{3.86} = P$

$10^{3.86} = 10^{3.74}(2)^k$ sub a, P & P_1 into model

$10^{3.86-3.74} = 2^k$

$k = \log_2 10^{0.27}$

$\boxed{k = 0.399}$ (3sf)

Q12B

b) If the model was used, P_1 would be negative. Since a will always be positive, this would always lead to a predicted loss. The model would suggest that the company never makes an annual profit.